18. Two solutions of $x^3y''' - 3xy' + 3y = 0 \quad (x > 0)$ are $\phi, (x) = x$, $\phi 2(n) = x^3$. Find a third independent solution.

19. Find a solution ϕ of the form $\phi(x) = x^r \sum_{k=0}^{\infty} C_k x^k$ for the equation $xy'' + (1-x)y' + \alpha y = 0$.

20. Consider the initial value problem $y' = 3y + 1$, $y(0) = 2$

(a) compute the first four approximations ϕ_0, ϕ_1, ϕ_2, ϕ_3

(b) Compute the solution $\phi(x)$.

NOVEMBER/DECEMBER 2024

GMA13/DMA13 — ORDINARY DIFFERENTIAL EQUATIONS

Time : Three hours Maximum : 75 marks

SHANMUGHA INDUSTRIES ARTS & SCIENCE COLLEGE LIBRARY

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define homogeneous equation with an example.

2. State Uniqueness theorem for the second order linear equation.

3. Find all real-valued solutions of $y'' + y = 0$.

4. State existence theorem for linear equations with constant coefficients.

5. Define Kironskian of $\phi_1, \phi_2 \ldots\ldots, \phi_n$.

6. Define the Legendre polynomial.

7. Write the Euler equation.

8. Write the Bessel function of order n of the first kind.

9. Find all real valued solutions of the equation $yy' = n$.

10. Check whether the equation $x^2y^3dn - x^3y^3dx = 0$ is exact or not.

ARTS & SCIENCE COLLEGE LIBRARY

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Find all solutions of the equation $y'' + (-1+3i)y' - 3iy = 0$.

Or

(b) Solve : $y'' - y' - 2y = e^{-x}$.

12. (a) Find all solutions of the equation $y^{(4)} - 16y = 0$.

Or

(b) Using the annihilator method find a particular solution of the equation $y'' + y = xe^x \cos 2x$

13. (a) Solve : $y'' + \frac{y'}{x} - \frac{y}{x^2} = 0$ with initial condition $y(1) = 1;\ y'(1) = 0$.

Or

(b) Show that $\int_{-1}^{1} P_n(x)P_m(x)dx = 0, (n \neq m)$.

14. (a) Find all the solutions of the equation $x^3y''' + 2x^2y'' - xy' + y = 0$, for $x > 0$.

Or

(b) Show that $x^{-\frac{1}{2}}\sqrt{\frac{1}{2}}(n) = \frac{\sqrt{2}}{\sqrt{\frac{1}{2}}}\cos n$

15. (a) Solve : $y' = \frac{x+y}{x-y}$.

Or

(b) Solve : $\cos x \cos y - 2\sin n \sin y dy = 0$

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Solve : $y'' + y = \tan x$.

17. Solve : $y'' - 4y' + 5y = 3e^{-x} + 2x^2$.